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Natural Convection Heat Transfer from Helicoidal Pipes

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An experimental investigation is reported on natural convection heat transfer in air from the outer surface of constant heat flux helicoidal pipes with vertical and horizontal orientations. The temperatures along the flow direction and peripheral direction of the tube wall were measured. The test Rayleigh numbers range from 4×10^3 to 1×10^5 for vertical coils and 5×10^3 to 1×10^5 for horizontal coils. The local and average Nusselt numbers are evaluated and correlated. For the vertical case, the results are compared with single horizontal cylinders. It is found that the heat transfer from the first turn is almost the same as that of the single horizontal cylinder. Because of the tube curvature, the heat transfer coefficient on the outer coil wall ($\psi = \pi/2$) is higher than that on the inner coil wall ($\psi = 3\pi/2$) in the middle turns of the coil. For the horizontal orientation, the results are well correlated with the tube diameter as the characteristic length. The local heat transfer characteristics are discussed as well. The overall average Nusselt number of the horizontal coil is higher than that of the vertical coil in the laminar region.

Nomenclature

b = pitch, m
D = coil diameter, m
d = tube diameter, m
F = shape factor
g = gravitational acceleration, ms^{-2}
H = coil height, m
h = heat transfer coefficient, $Wm^{-2} K^{-1}$
k = thermal conductivity, $Wm^{-1} K^{-1}$
N = turn number
Nu = Nusselt number
q = heat flux, Wm^{-2}
Ra = Rayleigh number
T = temperature, K
$\bar{T}$ = average temperature, K
α = thermal diffusivity, $m^2 s^{-1}$
β = isobaric thermal expansion coefficient, K^{-1}
ε = emissivity
ν = kinematic viscosity, $m^2 s^{-1}$
σ = Stefan–Boltzmann constant
ϕ = peripheral position on one turn of horizontal coil
ψ = peripheral angle of the tube cross section

Subscripts

c = convection
h = coil height
r = radiation
w = tube wall
ψ = peripheral local value
∞ = ambient

Introduction

As effective heat transfer equipment, helicoidal pipes are frequently used in numerous engineering fields, such as refrigeration, air conditioning, nuclear power engineering, and chemical engineering. The fluid flow and heat transfer inside the tube have been extensively investigated during past decades by many researchers.[1–4] In spite of their widespread use, there is little information available on natural convection outside such coils. Such information is essential to the use of helicoidal pipes in solar heating, storing technology, nuclear reactor safety, and the handling of nuclear waste.

The only reference on the subject problem found in the literature is an experimental investigation on steady-state turbulent natural convection heat transfer in water from vertical helically coiled tubes by Ali.[5] In his experiments, 10 coils were tested with four coil diameter-to-tube diameter ratios and five pitch-to-tube diameter ratios. The tube diameter of each coil was either 12 or 8 mm. Average heat transfer coefficients were obtained for turbulent natural convection in water. It was concluded that the heat transfer coefficient decreases with coil length for the coils with d = 12 mm, but increases with coil length for d = 8 mm. Three correlations for different tube diameters were suggested. For comparison, the correlation for d = 12 mm is cited:

$$Nu_H = 0.257 Ra_H^{0.323}, \quad 6 \times 10^8 \leq Ra_H \leq 3 \times 10^{11}, \quad d = 12\,\text{mm} \tag{1}$$

The much different behavior of the coils with different tube diameter is inexplicable.

Some studies about natural convection from single horizontal cylinders and vertical arrays of horizontal cylinders are cited and discussed for comparison, which are similar to the case of natural convection from the vertical helicoidal pipe. The investigations of natural convection heat transfer from long horizontal circular cylinders started as early as the beginning of this century.[6–8] One of the most important contributions of early studies is the simple model known as film theory for analyzing the convection heat transfer around cylinders.[7] In later years, several investigations and correlations were reported.[9–12] In 1975, after a comprehensive study and review of the existing experimental data, Morgan[13] proposed a set of correlations over a wide range of Rayleigh numbers $10^{-18} < Ra < 10^{12}$. The correlation that covers the present experimental range is

$$Nu = 0.480 Ra^{0.25}, \qquad 10^4 \leq Ra \leq 10^7 \tag{2}$$

Some useful correlations can also be found in Refs. 14–17. Recently, Clemes et al.[12] performed a new set of measurements

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on natural convection heat transfer in air from isothermal long horizontal cylinders with circular and noncircular cross sections. They were able to analyze and correlate their experimental data by using the conduction-layer approximate method discussed by Raithby and Hollands.[17,18]

Another related problem with considerable engineering applications is the natural convection from horizontal cylinders in an array set. For this configuration, the interaction between the temperature and flowfields around the neighboring cylinders can either reduce or enhance the average heat transfer coefficient relative to a single cylinder, depending on the specific configurations. Usually, heat transfer from the upper cylinder is degraded for small separation distances and enhanced for large ones.[19] Similar phenomena have been observed recently by Sadeghipour and Asheghi.[20] The effect of the lower cylinder on heat transfer to the upper cylinder is twofold. First, the plume from the lower cylinder supplies an initial velocity for the upper one and increases the intensity of flow turbulence. Second, the thermal wake of the lower cylinder makes the thermal boundary layer become thicker for the upper cylinder. The combination of these two effects determines the net effect of the lower cylinder on the heat transfer from the upper cylinder. For small separation distances, the second effect is dominant; however, for large separation distances, the first one is dominant. Eckert and Soehngen[21] performed experiments for three isothermal cylinders with a diameter of 22.3 mm. In the case of in-line arrangement for separation distance tested, it seems that heat transfer from the bottom cylinder remained the same as a single cylinder, and the heat transfer coefficient decreases from lower cylinder to higher cylinder as the plume develops.

However, it appears that data do not exist to compare the results of natural convection heat transfer with those of the horizontal helicoidal pipe.

Despite some similarities, the natural convection heat transfer from the vertical helicoidal coil is different from the vertical array of horizontal cylinders in two ways. First, the curvature and helix angle may make the temperature and flowfields around the tube tormented relative to a vertical array of horizontal cylinders. Second, the inside flow along the axis of the coil is somewhat similar to confined space natural convection instead of free space natural convection, especially for small coil diameter-to-tube diameter ratios and long coil height H. The flow and temperature fields of the natural convection from the horizontal coil seem to be more complex and truly three dimensional.

Experimental Measurement and Data Analysis

Test Setup and Experimental Procedures

Three helicoidal pipes were used as the test sections. Figure 1 shows a schematic view of the test section and thermocouple locations. The geometric parameters of these test sections are listed in Table 1.

The test section was installed in the center of a test room with dimensions of 1.8 × 1.8 × 1.8 m^3, which was in a larger lab room. On the bottom of the test room there was a 0.3-m slit around the four sides and the top was open. The test room was constructed using insulated foam plates with very low radiation emissivity surface to reduce the radiation heat loss from the test section. The temperature distribution inside the room was measured for three test cases with low, middle, and high Rayleigh numbers to see if the room was large enough as a free space, and the thermal stratification in the room can be neglected.

Twenty E-type thermocouples with a 0.1 mm diam were installed on the test section surface for the vertical cases of the first and second test section in such a way shown in Fig. 1a. There are 40 thermocouples for the third test section. Thirty-six thermocouples were used on the middle turn for the horizontal cases (Fig. 1b), to measure the temperature distribution along the turn and peripheral direction of the tube cross section. Six thermocouples around the test room wall were used to measure the ambient temperature. The thermocouples were calibrated against a precision thermometer and have an accuracy of 0.1°C. The first test section was filled with dry silicon sand of very low thermal conductivity of about 0.02 W/(m °C). Since the temperature gradient along the tube axis is not significant relative to the temperature difference between the tube surface and ambient, the conduction and convection inside of the tube can be neglected. The second and third test sections were highly vacuumized. The test section was heated by passing high dc current through the tube wall. Therefore, the boundary condition on the tube surface is thought to be uniform heat flux on the measured surfaces. Heat flux on the surface was determined by measuring the power input to the tube wall.

During the experiments, the test section was heated and kept undisturbed for 3–6 h, depending on the heating power, to reach a steady-state condition. The maximum variation of 0.2°C for each thermocouple reading during 20 min was set as the criterion of steady-state condition for most test cases. The temperatures were then measured four times to get the average values. All of the procedures were done automatically by an HP data acquisition system.

Table 1 Geometric parameters of the test sections

Test section	d, mm	D, mm	b, mm	No. of turns
1	25.4	259	62.5	5
2	12.7	127	28.2	5
3	12.7	127	76.0	10

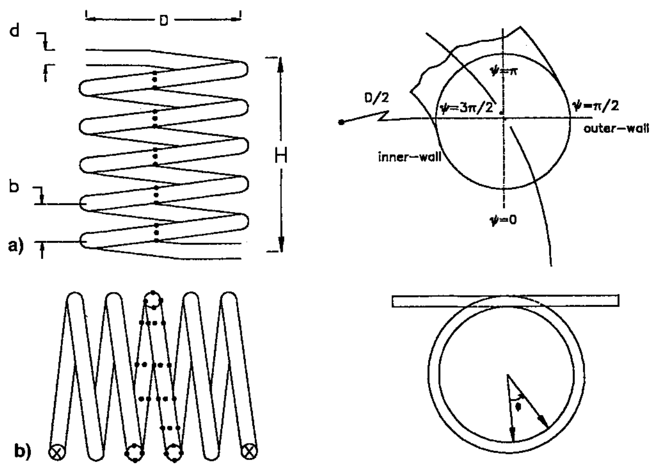

Fig. 1 Test section and thermocouple locations: a) vertical and b) horizontal.

Data Reduction

In this examination, the local and average Nusselt numbers as functions of the Rayleigh number were determined from the measured temperature distributions. The heat generated inside the tube wall dissipates from the coil surface by convection and radiation:

$$q_w = q_c + q_r \tag{3}$$

q_c and q_r are the fractions of the heat flux dissipating from the surface by convection and radiation, respectively, which can be determined by the following equations:

$$q_c = h(T_w - T_\infty) \tag{4}$$

$$q_r = \varepsilon F \sigma (T_w^4 - T_\infty^4) \tag{5}$$

The coefficient h can be calculated by

$$h = \frac{q_w - \varepsilon F\sigma(T_w^4 - T_\infty^4)}{(T_w - T_\infty)} \tag{6}$$

The Nusselt number is defined as

$$Nu = hd/k \tag{7}$$

In the previous equations, T_w is the temperature of the interested surface. Although Eq. (6) is written on a local basis, it can be used to get the average heat transfer coefficients by using the average wall temperature $\bar{T}_w$, since the temperature variation on the coil surface is very small, compared to the absolute temperature level. The local Nusselt number Nu_ψ at specific peripheral positions of each cross section is calculated from Eqs. (6) and (7), where the T_w takes the corresponding local temperature value.

The radiation heat transfer between the turns can be neglected as the small temperature differences among them. The shape factors between neighboring turns were estimated as the cylinder-to-cylinder shape factor with the same pitch and tube diameters. Then, the shape factors between the surface and surrounding were determined for the middle and end turns. The surface emissivity of the test section was estimated in the range of 0.5–0.6 for a commercial stainless steel tube pipe.[22] Because the wall of the test room has very low emissivity, the lower value of 0.5 was chosen for the calculation.

The results are presented as Nusselt number vs Rayleigh number. The Rayleigh number is defined as

$$Ra = \frac{g\beta d^3(\bar{T}_w - T_\infty)}{\upsilon\alpha} \tag{8}$$

The characteristic length is chosen as the o.d. of the tube. All of the thermal properties of air are based on the film temperature, the mean temperature of $\bar{T}_w$ and T_∞.

Based on the uncertainties of temperature (0.1°C), emissivity (5%), shape factor (5%), heating current (0.25%), voltage (0.25%), as well as those of geometric parameters and thermophysical properties, the uncertainties of the local Nusselt number, the average Nusselt number, and Rayleigh number were estimated as about 3.2–6.3, 2.8–5.2, and 1.2–3.0%, respectively, using the method recommended by Moffat.[23]

Results and Discussion

Heat Transfer from the Vertical Coil

In this section, the heat transfer from the first turn is presented and compared with that of a single horizontal cylinder. Then, the local Nusselt number distribution and the interaction between the neighboring turns are discussed. Finally, the overall average Nusselt numbers are correlated.

Heat Transfer from the First Turn

As the start point of the flow, the boundary layer is thinner on the first turn, and so the tube curvature effect is not significant. In this way, the heat transfer from the first turn is similar to the case of natural convection from a single horizontal cylinder. Figure 2 shows the average Nusselt number of the first turn vs the Rayleigh number for the first test section. The correlations of Merk and Prins[14] and Morgan[13] for single horizontal cylinders are also shown in the figure for comparison. These two correlations give the lowest and highest predictions in the test range for single horizontal cylinders among the correlations found in the literature. It is shown that the first turn results fall between these two correlations and near the Morgan correlation, which is based on experimental data rather than theoretical solution.[14] The symmetric distribution of peripheral Nusselt number distribution on the first turn (Fig. 3), also indicates that the heat transfer from the first turn is identical to the case of a single horizontal cylinder.

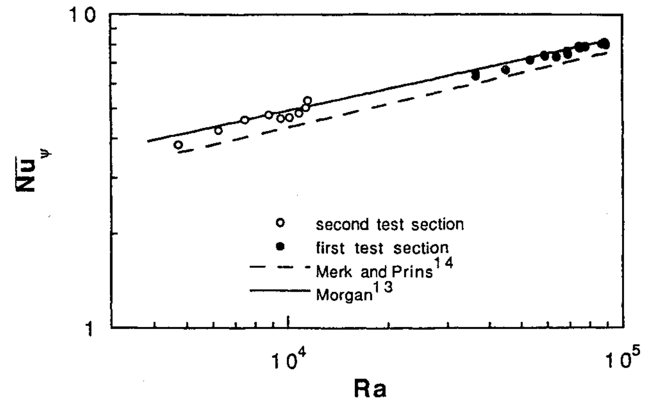

Fig. 2 Average Nusselt number of the first turn of the vertical coils and comparison with the correlations for a single horizontal cylinder.

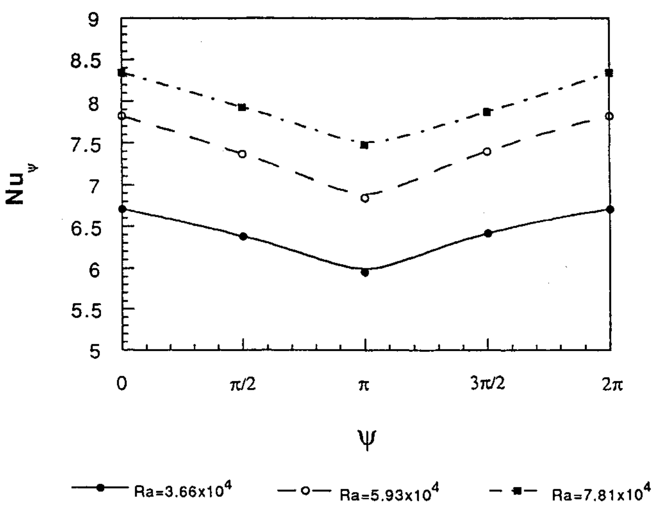

Fig. 3 Peripheral distribution of local Nusselt number of the first turn of the first test section in vertical orientation.

Local Nusselt Number Distribution

The peripheral Nusselt number distributions on the cross section of each turn of the first test section are plotted on Fig. 4. For each cross section, the thermal boundary layer develops from the bottom and becomes thickest on the top. Therefore, the local Nusselt numbers have the lowest value on the top and the highest value on the bottom, except on the fifth turn. The reason for the different distribution on the fifth turn may be that the plume converges to the axis of the coil above the top of the test section. Because of the curvature of the tube, the flow and temperature boundary layers are thicker on the inner side of the coil ($\psi = 3\pi/2$) than those on the outer side ($\psi = \pi/2$). Thus, the Nusselt number on the outer side is higher than that on the inner side at the third and fourth turns, where little end effect exists.

As discussed, the heat transfer from the first turn is almost the same as that from a single horizontal cylinder. As the plume develops, the heat transfer coefficient drops down on the second turn, then becomes higher on the third and fourth turn (Fig. 5). For the third test section, after the fifth turn, the Nusselt numbers increase and then decrease (Fig. 5c). This may be attributed to the transition of laminar flow to turbulent flow. Similar to the case of a vertical array of horizontal cylinders, the plume from the lower turn has two effects on the heat transfer of the upper turn. First, the plume makes the boundary layer become thicker on the next turn and degrades

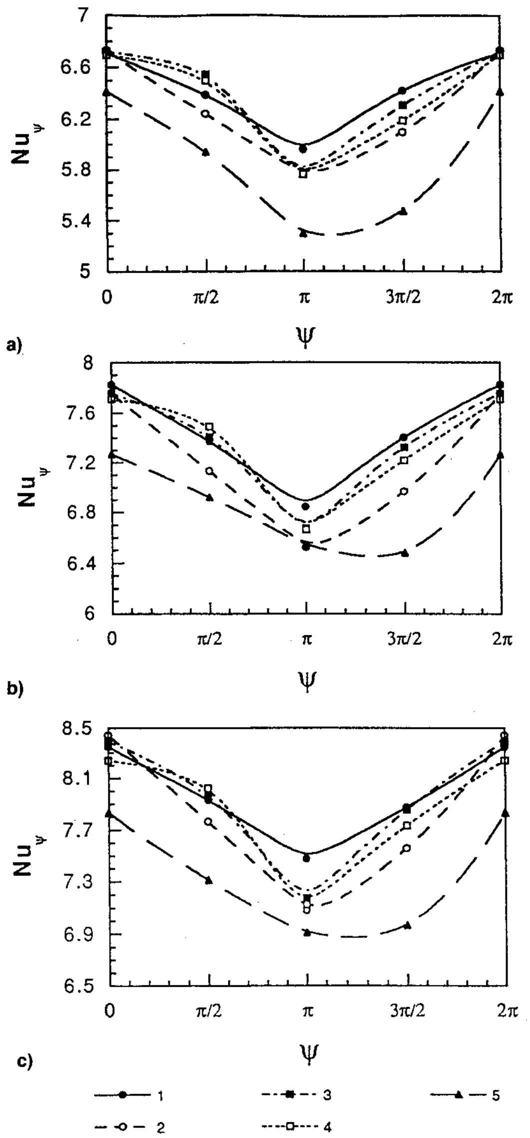

Fig. 4 Peripheral Nusselt number distribution of the first test section in vertical orientation: a) $Ra = 3.66 \times 10^4$, b) $Ra = 5.93 \times 10^4$, and c) $Ra = 7.81 \times 10^4$.

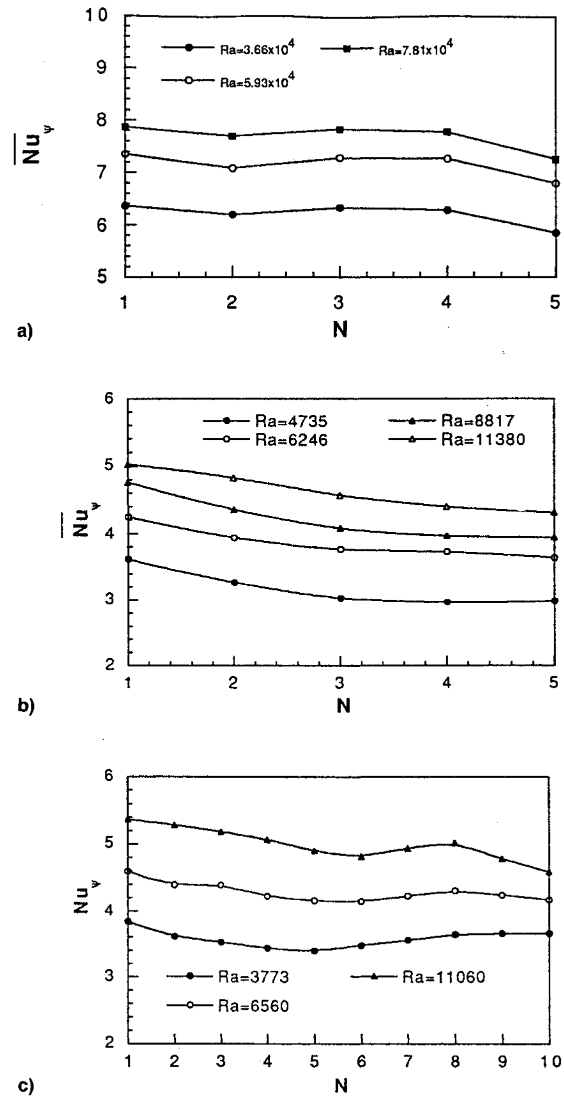

Fig. 5 Peripheral average Nusselt number variations along the flow direction of the vertical coils: a) first, b) second, and c) third test section.

the heat transfer. Second, the plume provides an initial velocity and increases the intensity of the flow turbulence for the next turn, which could improve the heat transfer from the upper turn. The first effect is dominant from first to second turn. The second effect is powerful from the second turn to the third and fourth turn. As the end effect on the top of the test section, the Nusselt number of the fifth turn has the lowest value for the first and second test sections (Figs. 5a and 5b). Figure 6 shows the local Nusselt number variations from the first to fifth turn at different peripheral positions for a typical case of $Ra = 3.66 \times 10^4$ of the first test section. It can be seen that the Nusselt numbers on the top ($\psi = \pi$) and bottom ($\psi = 0$) change gently. At the first turn, the Nusselt numbers on the outer side of the coil and inner side of the coil have the same value; but as the flow develops, the Nusselt number on the inner side is lower than that on the outer side, and the difference increases. This phenomenon comes from the fact that the boundary layer develops faster on the inner side of the coil because of the effect of the tube curvature.

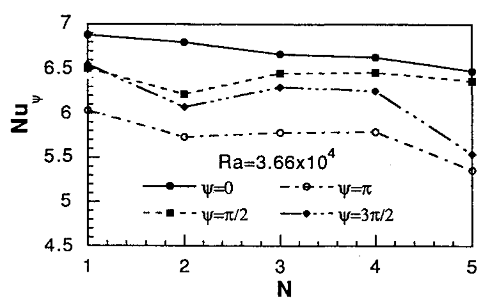

Fig. 6 Local Nusselt number variations along the flow direction of the first test section in vertical orientation.

Overall Average Nusselt Number

The overall average Nusselt numbers of the vertical coils for the total number of turns used in each experiment are plotted against the Rayleigh number in Fig. 7. It can be seen that the overall average Nusselt numbers of the third test section are higher than the other two. This can be explained as the flow may become turbulent at the upper portion of the third test section. The results of the first two test sections can be fitted into the following correlation:

$$Nu = 0.290Ra^{0.293}, \qquad 4 \times 10^3 \leq Ra \leq 1 \times 10^5 \qquad (9)$$

The maximum deviation between the experimental data and the correlation is 7.4%, except in the lowest Ra number case.

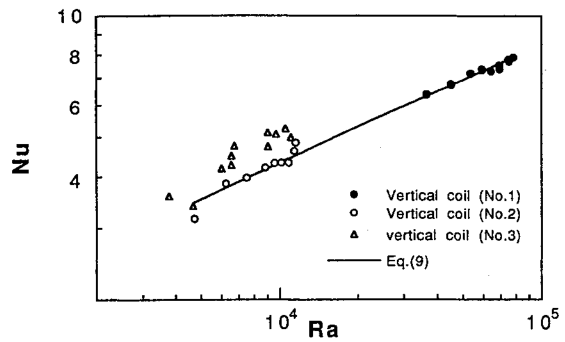

Fig. 7 Overall average Nusselt number vs Rayleigh number of the vertical coils.

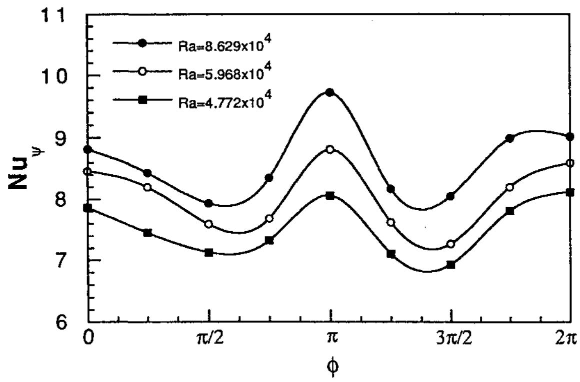

Fig. 8 Peripheral average Nusselt number distribution around the horizontal coil (first test section).

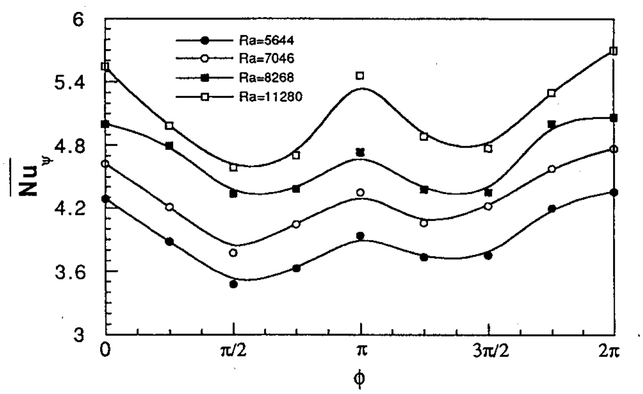

Fig. 9 Peripheral average Nusselt number distribution around the horizontal coil (second test section).

Heat Transfer from the Horizontal Coil

Local Nusselt Number Distribution

The flow and temperature fields of natural convection from the horizontal coil are extremely complex and definitely three dimensional. The peripheral Nusselt number distribution around the tube is different from one cross section to another. The peripheral variation of the local Nusselt number around the tube cross section ψ is much smaller compared to the variation around the coil ϕ. In the middle turns of the coil, the Nusselt number distribution along the coil should be periodical. In this experiment, the temperature distribution on the third turn of the coil was measured. The peripheral average Nusselt number $\overline{Nu}_\psi$ distribution around the turn is shown in Figs. 8 and 9 for the first and second test sections. The $\overline{Nu}_\psi$ is defined as the peripheral average Nusselt number for each cross section. It is shown that the heat transfer is stronger on the top ($\phi = \pi$) and the bottom ($\phi = 0$ or 2π) than on either side ($\phi = \pi/2$ and $3\pi/2$). As the Rayleigh number increases, the flow becomes stronger, therefore, the peripheral average

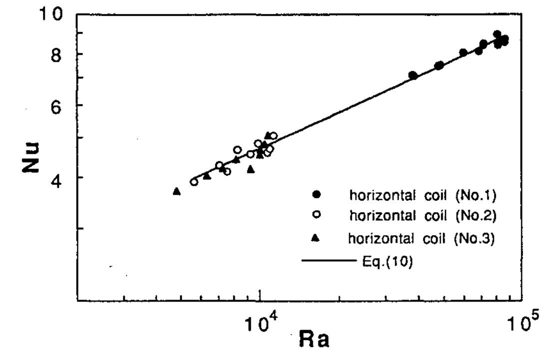

Fig. 10 Overall average Nusselt number vs Rayleigh number of the horizontal coils.

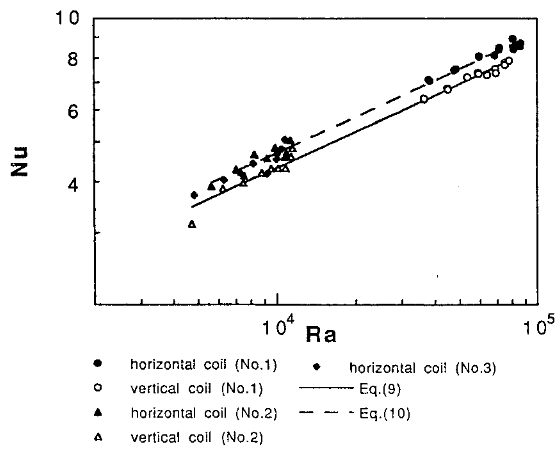

Fig. 11 Comparison of the overall average Nusselt number between the vertical and horizontal coils.

Nusselt number variation around the turn becomes more significant. The same behavior was observed for the third test section.

Overall Average Nusselt Number

The overall average Nusselt number of the middle turns of the horizontal coil is shown in Fig. 10 vs the Rayleigh number. The following correlation was obtained from the results:

$$Nu = 0.318Ra^{0.293}, \quad 5 \times 10^3 \leq Ra \leq 1 \times 10^5 \quad (10)$$

The maximum deviation between the experimental data and correlation is 4.8%.

When comparing Eq. (9) with Eq. (10), it seems that the average heat transfer coefficient of the horizontal coil is about 10% higher than that of the vertical coil in the laminar region (Fig. 11).

Summary and Conclusions

The local and average heat transfers were measured for natural convection from the outside of vertical and horizontal helicoidal pipes. For the vertical orientation, heat transfer from the first turn is similar to the case of a single horizontal straight cylinder. The heat transfer from the last turn was affected significantly by the convergence of the plume on the top of the test section. For each cross section in the first four turns, the local Nusselt number has the lowest value on the top and the highest value on the bottom, caused by the boundary layer developing around the tube cross section. Because of the effect of tube curvature, the Nusselt number on the outer side of the coil is higher than that on the inner side at the middle turns. In the laminar region, the correlation of the overall average Nusselt number was suggested [Eq. (9)]. For the horizontal case, the correlation of the overall average Nusselt number is given [Eq. (10)]. The maximum deviation from the experimental data is about 7.4 and 4.8%, respectively. It has to be noted that Eqs. (9) and (10) are for the laminar natural convection on the surface of helicoidal pipes in air. In the experiment, the curvature ratio d/D is near 0.1, and the tube diameter changes from 12.7 to 25.4 mm.

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